SUPPLEMENTARY NOTES ON THE REMOTE CONTROL OF ELECTRICAL STIMULATION OF THE NERVOUS SYSTEM*

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In the original paper† the apparatus for the remote control of nerve stimulation was described and the theory of action was presented. Although the analysis of the electrical system was simplified to a considerable degree by making certain approximations, yet, because of the many factors upon which its operation depends, the design of a system at all different from the specific one described would require considerable thought and tedious calculation. Since the publication of the original article a number of practical questions have arisen involving certain changes in the sizes of some of the elements. An example of such questions is the following: how much larger can the primary coil be made so as to accommodate a larger cage or more than one cage and give a peak potential of ten volts across the stimulating coil? The following notes are presented to facilitate answering such questions and are hence directed to those who may be interested in setting up a system of the type described in the original paper.

The peak value of the current, I_2', through the tissue being stimulated is given approximately by Eq. (27) of the original paper. This equation is reproduced below.

$$I_2' = \frac{M(E_1 - E_s)}{R_2' L_1 \sqrt{\left(1+\frac{R_2}{R_2'}\right)^2 + \left(\frac{L_2\omega_1}{R_2'}\right)^2\left(1-\frac{L_1C_1}{L_2C_2'}\right)^2}} \text{ (approx)}, \qquad (27)$$

where M is the mutual inductance between the primary and second-

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ary coils; E_1 is the voltage to which the primary condenser is charged; E_S is 14 volts; R_2' is the resistance of the conduction path through the tissue between the stimulating electrodes; L_1 and C_1 are the inductance and capacitance of the primary circuit; L_2 is the inductance of the secondary coil; R_2 is the resistance of the secondary coil; C_2' is the polarization capacitance considered in series with R_2'; and $\omega_1 = \frac{2\pi}{T_1'}$, is approximately equal to $\frac{1}{\sqrt{L_1C_1}}$. T_1' is the time duration of the current pulse in the primary circuit and is approximately equal to the time duration of the secondary current pulse.

The product $I_2'R_2'$ ($= E_2'$) is approximately equal to the voltage between the stimulating electrode and the passive electrode. If it be assumed that the specific resistance of the tissue being stimulated is more or less definite and constant, whereas R_2' depends upon the area of the stimulating electrode in contact with the tissue, the major part of R_2' being undoubtedly concentrated in a small volume immediately surrounding the stimulating electrode, then the current density at the stimulating electrode is proportional to E_2', whatever the area of the electrode may be. Hence E_2' is a better measure of the current density than is I_2'. Presumably the *intensity* of stimulation of any small volume of nervous tissue is dependent upon the current density through the small volume, whereas the number of nerve fibers stimulated depends upon the volume surrounding the stimulating electrode in which the current density is above the threshold value for stimulation. The *quantity* of stimulation, if that term may be used to indicate the number of nerve fibres stimulated, depends therefore upon the area of the electrode as well as upon the current density. It is assumed in these notes that intensity of stimulation is more fundamental than quantity of stimulation, and hence E_2' is used as a measure of the stimulation produced rather than I_2'.

Experiments have shown that the quantity $R_2'C_2'\omega$ is practically constant for electrolytic conduction, and from measurements made on electrodes in contact with the brain tissue of a monkey, the value of this quantity is approximately 3 but may possibly be as high as 4 or 5 under different conditions. If this constant be denoted by K, R_2' in Eq. (27) can be replaced by $\frac{K}{C_2'\omega_1}$, and Eq. (27) reduces to

$$\frac{E_2'}{E_1 - E_s} = \frac{M}{L_1\sqrt{\left(1+\frac{R_2}{R_2'}\right)^2 + \frac{1}{K^2}\left(L_2 C_2' \omega_1^2 - 1\right)^2}} \qquad (40)^*$$

If in Eq. (40) $L_2C_2'\omega_1^2$ is made equal to unity the voltage ratio $\frac{E_2'}{E_1 - E_S}$ is practically independent of C_2', and also of R_2' since the ratio $\frac{R_2}{R_2'}$ is of the order of 0.06 and for a rough evaluation of the voltage ratio, can be neglected. Furthermore, by thus making the second bracket in the radical negligible the voltage ratio is made large. Then Eq. (40) reduces to the simple form

$$\frac{E_2'}{E_1 - E_s} \approx \frac{M}{L_1} \qquad (41)$$

and is dependent only upon the geometry of the primary and secondary coils. Using this simple relation, the design of the apparatus is very much simplified and, as will now be shown, the sizes of the elements to meet prescribed conditions can be found from a few simple charts.

The value of M is given by Eq. (23) and depends upon the number of turns in the primary and secondary coils and upon their dimensions. If L_1 is wound with copper ribbon, the inductance is given accurately enough by Maxwell's formula. This formula is given in Eq. (42).

$$L_1 = \frac{4\pi r_1 N_1^2}{10^3}\left(\log_e \frac{8 r_1}{0.2235\,(b+c)} - 2\right) \text{ microhenrys} \qquad (42)$$

where r_1 is the radius of the coil in centimeters, b is the width of the copper ribbon, and c is the radial depth of the winding. Combining equations (23) and (42) gives

$$\frac{E_2'}{E_1 - E_s} = \frac{M}{L_1} = \frac{\pi\left(r_{20}^2 + r_{20} r_{21} + r_{21}^2\right)}{6 r_1^2\left(\log_e \frac{8 r_1}{0.2235\,(b+c)} - 2\right)} \cdot \frac{N_2}{N_1} \qquad (43)$$

where r_{20} and r_{21} are the outside and inside radii of the secondary coil.

Equation (42) is plotted against r_1 for various values of N_1 in the full-line curves of Fig. 16 for a circular coil wound with 1″ copper ribbon. The dotted curves of Fig. 16 are Eq. (43) plotted

* The new equations and figures in this paper are consecutively numbered with the equations and figures in the original article.

for various values of $\frac{N_2}{N_1}$. These dotted curves are for a particular size of secondary coil shown in Fig. 2 and described on page 94 of the original article. Curves applying to any other size of secondary coil can be calculated by Eq. (43). The scale for $\frac{N_2}{N_1}$ shown at the right of Fig. 16 is a logarithmic scale which permits interpolation between the particular values of $\frac{N_2}{N_1}$ for which the curves are drawn, these values being the readings on the scale at the end of the curves. The use of Fig. 16 will be further explained later.

The inductance of the secondary coil is proportional to N_2^2 and, for the particular size described, is given by the following expression

$$L_2 = 0.0425 \times 10^{-6} N_2^2 \text{ henrys} \tag{44}$$

As has been stated, the condition that Eq. (40) reduces to Eq. (41) is that

$$L_2 C_2' \omega_1^2 = 1 \tag{45}$$

Furthermore, the dotted curves of Fig. 16 were calculated under the condition that Eq. (45) is fulfilled. In order easily to determine corresponding values of L_2, C_2', and n_1 $(= \frac{\omega_1}{2\pi})$ which satisfy Eq. (45), Fig. 17 is useful. Values of C_2' and L_2 are read on the second and fifth scales, respectively. A line connecting specific values of C_2' and L_2 intersects the central scale at a frequency which satisfies Eq. (45). For example, if C_2' is 0.05 μf and L_2 is 0.17 henry, the corresponding frequency as given by the intersection of the dashed line and the central scale is 1700 cycles per second. The duration of the secondary pulse would then be approximately $\frac{1}{1700}$ second. The fourth scale gives the number of turns of the particular size of secondary coil described, having inductance given on the fifth scale.

Similarly, the two outer scales give values of C_1 and L_1. These may be connected by a line which intersects the central scale at a frequency which satisfies the relation

$$L_1 C_1 \omega_1^2 = 1 \tag{46}$$

a relation which is approximately true, as previously pointed out.

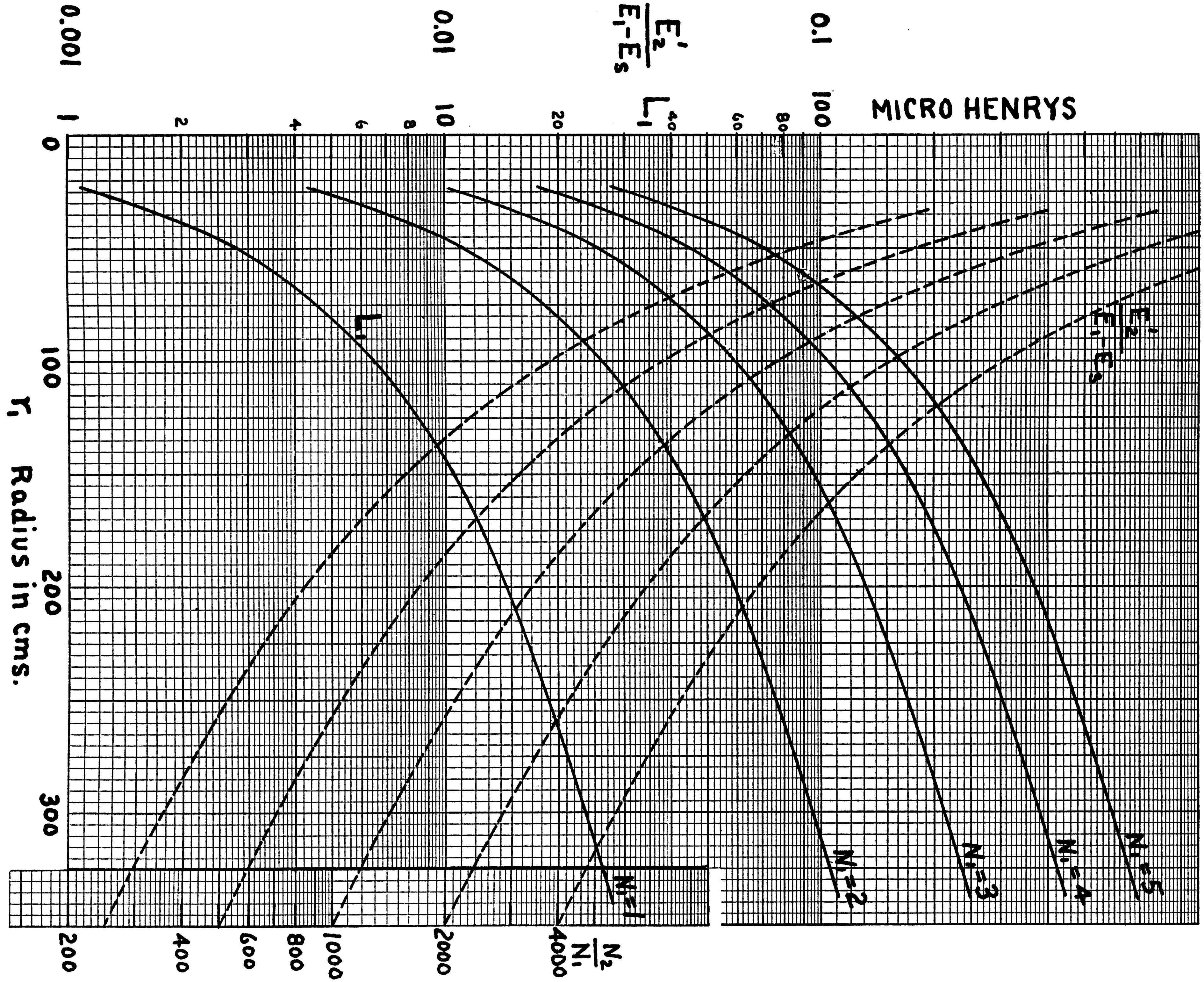
MICRO HENRYS
$\frac{E_2'}{E_1-E_s}$
L_1
r_1 Radius in cms.
$\frac{N_2}{N_1}$
$N_1=1$
$N_1=2$
$N_1=3$
$N_1=4$
$N_1=5$
0.001
0.01
0.1
0
100
200
300
1
2
4
6
8
10
20
40
60
80
100
200
400
600
800
1000
2000
4000

Fig. 16.

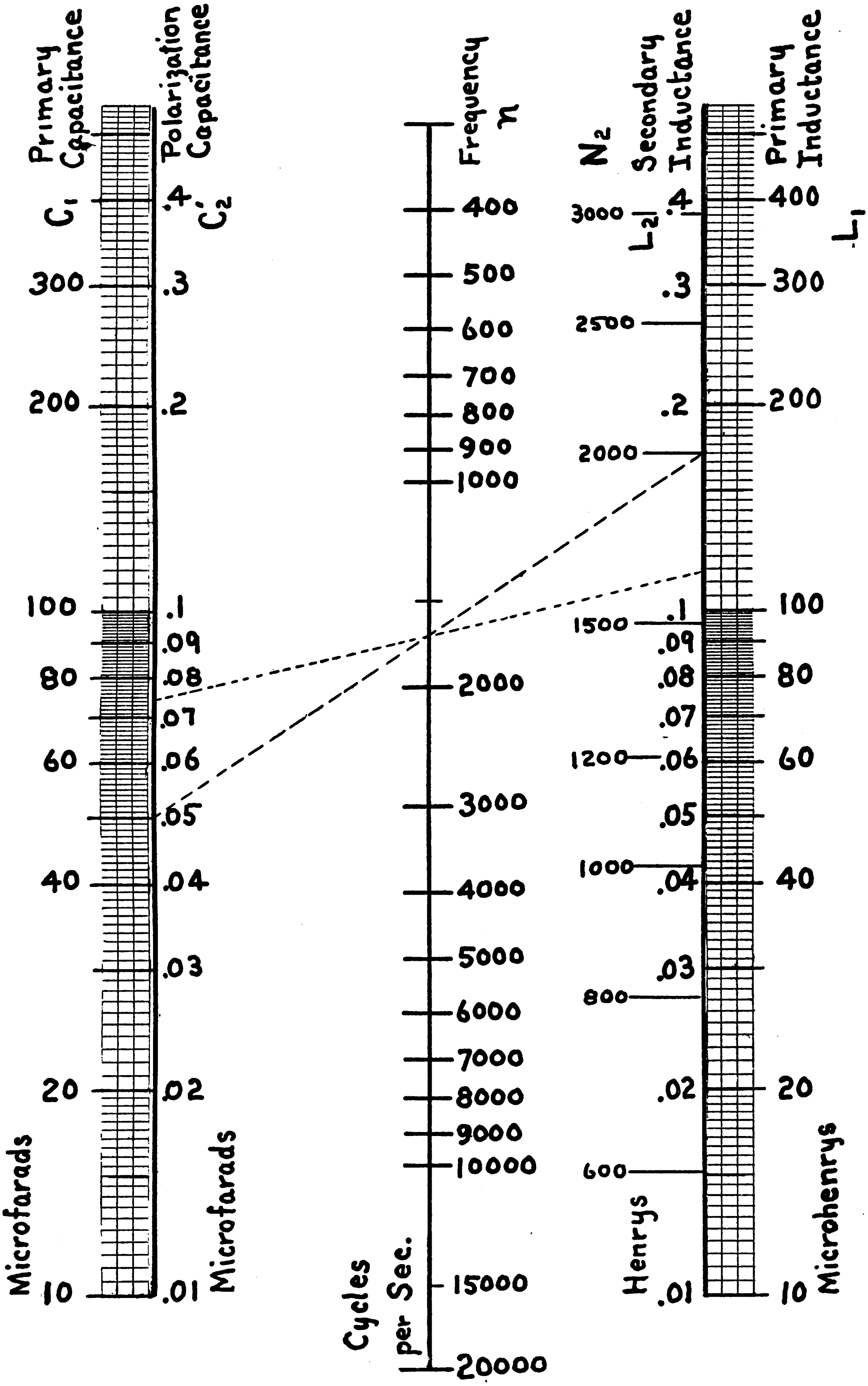
Primary Capacitance
C_1
Polarization Capacitance
C_2'
Microfarads
10
20
40
60
80
100
200
300
Microfarads
.01
.02
.03
.04
.05
.06
.07
.08
.09
.1
.2
.3
.4
Frequency
n
400
500
600
700
800
900
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
15000
20000
Cycles per Sec.
N_2
Secondary Inductance
L_2
3000
2500
2000
1500
1200
1000
800
600
Henrys
.4
.3
.2
.1
.09
.08
.07
.06
.05
.04
.03
.02
.01
Primary Inductance
L_1
400
300
200
100
80
60
40
20
10
Microhenrys

Fig. 17.

The use of Figs. 16 and 17 will now be explained by two specific examples.

Example 1. Assume that the secondary coil has 2000 turns and is used with electrodes of No. 32 wire. Suppose it is desired to know what size primary condenser should be used with a primary coil of 5 turns and radius 82.5 cms. (32.5 inches), and what maximum voltage would then be produced across the electrodes if the primary condenser were charged to 500 volts. The secondary inductance, L_2 is, by Eq. (44), equal to 0.17 henry, and C_2' is by experiment about 0.05μf. From Fig. 17, the frequency n_1 which satisfies Eq. (45) is 1700. The full-line curve for $N_1 = 5$ of Fig. 16 gives for L_1 the value 115 microhenrys. Fig. 17 shows that the primary capacitance, C_1, should be 74 microfarads. Referring again to Fig. 16 to find the peak voltage, E_2', which will exist between the stimulating electrodes, one finds the intersection of a vertical line for r_1 equal to 82.5 cms. and a dotted line corresponding to $\frac{N_2}{N_1}$ equal to $\frac{2000}{5}$, which gives $\frac{E_2'}{E_1 - E_S}$ equal to 0.046, or $E_2' = 0.046(500 - 14) = 22$ volts.

Example 2. Suppose it be required to find the largest primary coil which will give a peak stimulating voltage of 10 when the secondary coil has 2000 turns and is provided with electrodes of No. 32 wire. Let $E_1 - E$ be 400 volts.

Fig. 17 gives a frequency of 1700 to satisfy Eq. (45). The value of $\frac{E_2'}{E_1 - E_S}$ corresponding to the data given above is 0.025. Following along a horizontal line at 0.025 in Fig. 16 gives the following possibilities for r_1 according to the number of turns in the primary coil. (See Table 1.)

TABLE 1

N_1	N_2/N_1	r_1	L_1	C_1
1	2000	232 cms	17.3 μh	680 μf
2	1000	167 "	48.0 "	177 "
3	666	137 "	87.0 "	98 "
4	500	121 "	116.0 "	73 "
5	400	110 "	143.0 "	59 "

Figure 16 also gives the inductance L_1 recorded in the third column. The values of C_1 to fulfill Eq. (46) as given by Fig. 17 are recorded in the last column of Table 1.

In the calculations given above it has been assumed that Eq. (45) was rigidly fulfilled. This is not necessary and the method of calculation when there is a departure from Eq. (45) will now be given. Let

$$L_2 C_2' \omega_1^2 = P \tag{47}$$

represent the amount of departure where P is a factor usually ranging from unity to a value of two or three. If P differs from unity, Eq. (40) does not reduce to Eq. (41) and it is necessary to consider the value of the radical in Eq. (40). Let the value of the radical be denoted by F. Neglecting $\frac{R_2}{R_2'}$ in comparison with unity, and assuming K to be 3, F has the values shown plotted against P in Fig. 18. If P has a value different from unity, the voltage ratio $\frac{E_2'}{E_1 - E_S}$ is that when P equals unity divided by F. For example, if P is 2.5 then F is 2 and the voltage ratio is one-half that for P equal to unity. Now P may to advantage be made greater than unity by rasing ω_1 above the resonance value for the secondary circuit, thereby requiring a smaller value of C_1 or of L_1 than required for resonance.

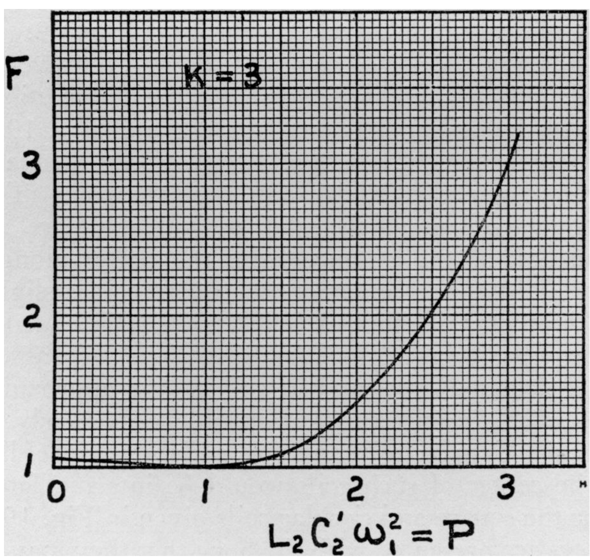

FIG. 18.

For illustration refer to the condition for N_1 of 2 in Table 1. The capacitance C_1 required if P is unity is 177 μf and the frequency n_1 is 1700 cycles per second. Suppose P is made equal to 2.5 by increasing n_1 to $1700\sqrt{2.5}$ or 2690 cycles per second. Then the stimulating voltage would be reduced to 1/2 of the previous value or 5 volts, and the value of C_1 required is about 72 μf instead of 177 μf. This value 72 is obtained from Fig. 17 by drawing a line from $L_1 = 48$ μh through $n_1 = 2690$ and noting the value of C_1 given by the intersection of this line and the left-hand scale.

The primary coil in the analysis given above is assumed to be a circular coil. When the coil is large it is easier from a practical standpoint to construct a square or an octagonal coil. The question naturally arises as to what effect does changing the shape of the coil have upon the voltage produced and the inductance of the coil.

The inductance of a square coil wound with copper ribbon is given by the expression

$$L_1 = \frac{8l}{10^3}\left(\log_\varepsilon \frac{l}{b+c} + 0.2235 \cdot \frac{b+c}{l} + 0.726\right) \text{microhenrys}, \quad (48)$$

where l is the length of a side, and b and c have the same significance as for Eq. (42). *If the radius r_1 of the circular coil is equal to 0.60 of the length l of a side of the square, the two coils have the same inductance.* The relative sizes of the two coils having the same inductance are shown in the upper left corner of Fig. 19.

Two coils having the same inductance do not, however, have the same field strength at the center nor the same distribution of field over the area of the coils. The full-line curve of Fig. 19 gives the field for unit current in the plane of a circular coil of one turn as a function of the distance from the center of the coil. The dotted curves give the variation of field in an equivalent square coil of one turn as a function of the distance from the center along lines *oa* and *ob*. The field at the center of the square coil is slightly more than that in the center of the round coil in the ratio of 1.08, but the field is slightly less uniform with the square coil, as shown by Fig. 19. A hexagonal or octagonal coil having approximately the same perimeter as the circular coil would have nearly the same inductance as the circular coil, and the variation of field with distance from the center of such coils would be intermediate between the curves for the square and circular coils given in Fig. 19.

These notes are presented with the hope that they may save some experimenters in this field a considerable amount of time in deciding upon a design of the stimulating apparatus. Reference to the original article where the complete theory of the electrical circuits is given shows that an accurate calculation of the potential or current produced in the secondary coil is very tedious and in general would be a waste of time because usually the values of some of the factors

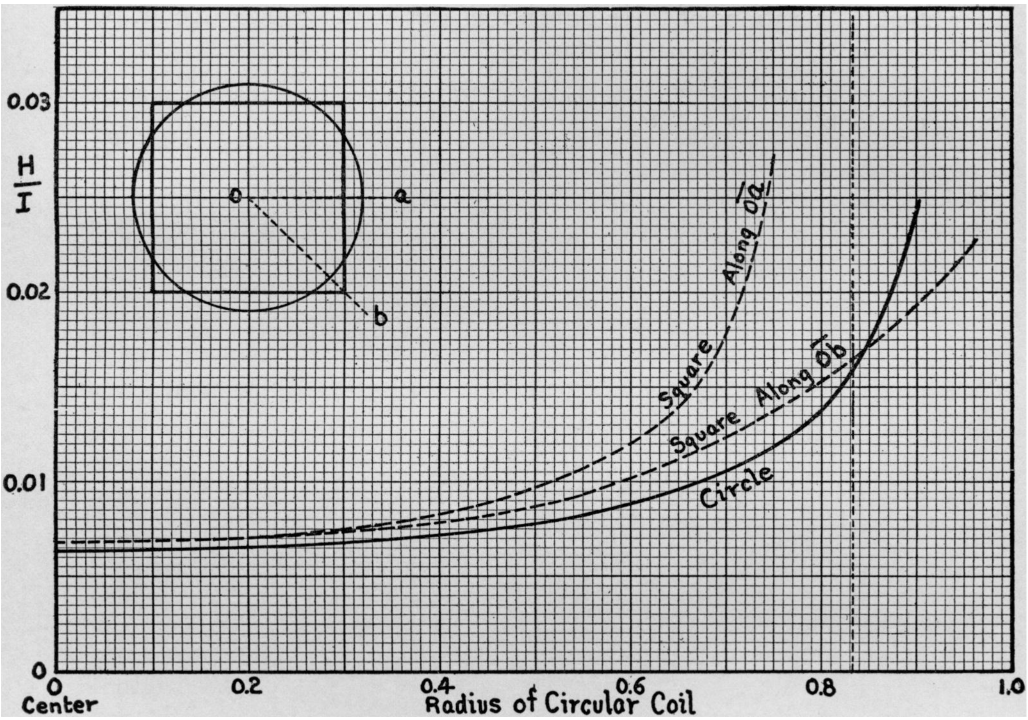

FIG. 19.

are not accurately known. Certain approximations which have been made are fully justified when the sizes of the various elements are somewhere near to those used in the specific arrangement of Dr. Light's. If the experimental arrangement differs greatly from that described, these approximations must be re-examined. Only by making further approximations, some of which may cause errors of ten per cent or more, is it possible to reduce the analysis to the simple expressions given above. It is emphasized, therefore, that the methods outlined in this paper are to be considerd as rough guides only. Even if all of the factors concerned in the approximate expressions given in this paper were accurately known, the voltage E_2' calculated by the charts and formulae here given might be in error by ten or fifteen per cent. As has been pointed out, Fig. 16 and the scale of N_2 of Fig. 17 are valid only for the particular shape and size of secondary coil, and when the primary coil is wound with ribbon. Variations of these dimensions can be taken into account by referring to the original formulae.

Errata: In the original article, on page 116, wherever R'_2 appears multiplied by C_2, the R'_2 should read R'^2_2.